Lesson 11: Conditional Relative Frequencies and Association

After further discussion, the students involved in designing the superhero comic strip decided that before any decision is made, a more careful look at the data on the special powers a superhero character could possess was needed. There is an association between gender and superpower response if the superpower responses of males are not the same as the superpower responses of females. Examining each row of the table can help determine whether or not there is an association.

Classwork

Example 1: Conditional Relative Frequencies

Recall the two-way table from the previous lesson:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **To fly** | **Freeze time** | **Invisibility** | **Super Strength** | **Telepathy** | **Total** |
| **Females** | 49 | 60 | 48 | 1 | 70 | 228 |
| **Males** | 51 | 71 | 27 | 25 | 48 | 222 |
| **Total** | 100 | 131 | 75 | 26 | 118 | 450 |

A **conditional relative frequency** compares a frequency count to the marginal total that represents the condition of interest. For example, the condition of interest in the first row is females. The row conditional relative frequency of females responding “Invisibility” as the favorite superpower is 48/228 or approximately 0.211. This conditional relative frequency indicates that approximately 21.1% of females prefer “Invisibility” as their favorite superpower. Similarly, 27/222, or approximately 0.122 or 12.2%, of males prefer “Invisibility” as their favorite superpower.

Exercises 1–5

1. Use the frequency counts from the table in Example 1 to calculate the missing row conditional relative frequencies. Round the answers to the nearest thousandth.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **To Fly** | **Freeze Time** | **Invisibility** | **Super Strength** | **Telepathy** | **Total** |
| **Females** |  |  | $\frac{48}{228}$ ≈ 0.211 |  |  |  |
| **Males** | $\frac{51}{222}$  0.230 |  |  |  |  | $\frac{222}{222}$ = 1.000 |
| **Total** |  |  | $\frac{75}{450}$ ≈ 0.167 |  |  |  |
|  |  |  |  |  |  |  |

2. Suppose that a student is selected at random from those who completed the survey. What do you think is the

 gender of the student selected? What would you predict for this student’s response to the superpower question?

3. Suppose that a student is selected at random from those who completed the survey. If the selected student is male, what do you think was his response to the selection of a favorite superpower? Explain your answer.

1. Suppose that a student is selected at random from those who completed the survey. If the selected student is female, what do you think was her response to the selection of a favorite superpower? Explain your answer.
2. What superpower was selected by approximately one-third of the females? What superpower was selected by approximately one-third of the males? How did you determine each answer from the conditional relative frequency table?

Example 2: Possible Association Based on Conditional Relative Frequencies

Two categorical variables are associated if the row conditional relative frequencies (or column relative frequencies) are different for the rows (or columns) of the table. For example, if the selection of superpower selected for females is different than the selection of superpowers for males, then gender and superpower favorites are associated. This difference indicates that knowing the gender of a person in the sample indicates something about their superpower preference.

The evidence of an association is strongest when the conditional relative frequencies are quite different. If the conditional relative frequencies are nearly equal for all categories, then there is probably not an association between variables.

Exercises 6–10

Examine the conditional relative frequencies in the two-way table of conditional relative frequencies you created in Exercise 1. Note that for each superpower, the conditional relative frequencies are different for females and males.

1. For what superpowers would you say that the conditional relative frequencies for females and males are very different?
2. For what superpowers are the conditional relative frequencies nearly equal for males and females?
3. Suppose a student is selected at random from the students who completed the survey. If you had to predict which superpower this student selected, would it be helpful to know the student’s gender? Explain your answer.
4. Is there evidence of an association between gender and superpower selected? Explain why or why not.
5. What superpower would you recommend the students at Rufus King High School select for their superhero character? Justify your choice.

Example 3: Association and Cause-and-Effect

Students were given the opportunity to prepare for a college placement test in mathematics by taking a review course. Not all students took advantage of this opportunity. The following results were obtained from a random sample of students who took the placement test:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Placed in Math 200** | **Placed in Math 100** | **Placed in Math 50** | **Total** |
| **Took Review Course** | 40 | 13 | 7 | 60 |
| **Did not take Review Course** | 10 | 15 | 15 | 40 |
| **Total** | 50 | 28 | 22 | 100 |

Exercises 11–16

1. Construct a row conditional relative frequency table of the above data.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Placed in Math 200** | **Placed in Math 100** | **Placed in Math 50** | **Total** |
| **Took review course** |  |  |  |  |
| **Did not take review course** |  |  |  |  |
| **Total** |  |  |  |  |
|  |  |  |  |  |

1. Based on the conditional relative frequencies, is there evidence of an association between whether or not a student takes the review course and the math course in which the student was placed? Explain your answer.
2. Looking at the conditional relative frequencies, the proportion of students who placed into Math 200 is much higher for those who took the review course than for those who did not. One possible explanation is that taking the review course caused improvement in placement test scores. What is another possible explanation?

Now consider the following statistical study:

Fifty students were selected at random from students at a large middle school. Each of these students was classified according to sugar consumption (high or low) and exercise level (high or low). The resulting data are summarized in the following frequency table.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | **Exercise Level** |  |
|  |  | **High** | **Low** | **Total** |
| **Sugar Consumption** | **High** | 14 | 18 | 32 |
| **Low** | 14 | 4 | 18 |
|  | **Total** | 28 | 22 | 50 |

14. Calculate the row conditional relative frequencies, and display them in a row conditional relative frequency table.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | **Exercise Level** |  |
|  |  | **High** | **Low** | **Total** |
| **Sugar Consumption** | **High** |  |  |  |
| **Low** |  |  |  |
|  | **Total** |  |  |  |

15. Is there evidence of an association between sugar consumption category and exercise level? Support your answer using conditional relative frequencies.

16. Do you think it is reasonable to conclude that high sugar consumption is the cause of the observed differences in the conditional relative frequencies? What other explanations could explain a difference in the conditional relative frequencies? Explain your answer.

Lesson Summary

* A conditional relative frequency compares a frequency count to the marginal total that represents the *condition* of interest.
* The differences in conditional relative frequencies are used to assess whether or not there is an association between two categorical variables.
* The greater the differences in the conditional relative frequencies, the stronger the evidence that an association exits.
* An observed association between two variables does not necessarily mean that there is a cause-and-effect relationship between the two variables.

Problem Set

1. Juniors and seniors were asked if they plan to attend college immediately after graduation, seek full-time employment, or choose some other option. A random sample of 100 students was selected from those who completed the survey. Scott started to calculate the row conditional relative frequencies to the nearest thousandth.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Plan to Attend College** | **Plan to seek Full-Time Employment** | **Other Options** | **Totals** |
| **Seniors** | $$\frac{25}{55} ≈ 0.455$$ | $$\frac{10}{55} ≈ 0.182$$ | $$\frac{20}{ } ≈ $$ | $$\frac{55}{55} = 1.000$$ |
| **Juniors** | $$\frac{35}{ } ≈ $$ | $$\frac{5}{ } ≈ $$ | $$\frac{5}{45} ≈ 0.111$$ | $$\frac{45}{45} = 1.000$$ |
| **Totals** | $$\frac{60}{100} = 0.600$$ | $$\frac{15}{100} = 0.150$$ |  | $$\frac{100}{100} = 1.000$$ |

2. Complete the calculations of the row conditional relative frequencies. Round your answers to the nearest thousandth.

3. Are the row conditional relative frequencies for juniors and seniors similar, or are they very different?

4. Do you think there is a possible association between grade level (junior or senior) and after high school plan? Explain your answer.

5.Consider again the summary of data from the 100 randomly selected students in the Rufus King investigation of after-school activities and gender:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Intramural Basketball** | **Chess Club** | **Jazz Band** | **Not involved** | **Total** |
| **Females** | 20 | 10 | 10 | 20 | 60 |
| **Males** | 20 | 2 | 8 | 10 | 40 |
| **Total** | 40 | 12 | 18 | 30 | 100 |

6. Construct a row conditional relative frequency table for this data. Decimal values should be given to the nearest thousandth.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Intramural Basketball** | **Chess Club** | **Jazz Band** | **Not involved** | **Total** |
| **Females** |  |  |  | $$\frac{20}{60}=0.333$$ |  |
| **Males** |  | $$\frac{2}{40}=0.050$$ |  |  |  |
| **Total** |  |  |  |  |  |

1. For what after-school activities do you think the row conditional relative frequencies for females and males are very different? What might explain why males or females select different activities?

1. If John, a male student at Rufus King High School, completed the after-school survey, what would you predict was his response? Explain your answer.
2. If Beth, a female student at Rufus King High School, completed the after-school survey, what would you predict was her response? Explain your answer.
3. Notice that 20 female students participate in intramural basketball and that 20 male students participate in intramural basketball. Is it accurate to say that females and males are equally involved in intramural basketball? Explain your answer.
4. Do you think there is an association between gender and choice of after-school program? Explain.

**Column conditional relative frequencies** can also be computed by dividing each frequency in a frequency table by the corresponding column total to create a column conditional relative frequency table. Column conditional relative frequencies indicate the proportions or relative frequencies based on the column totals.

1. If you wanted to know the relative frequency of females surveyed who participated in chess club, would you use a row conditional relative frequency or a column conditional relative frequency?
2. If you wanted to know the relative frequency of band members surveyed who were female, would you use a row conditional relative frequency or a column conditional relative frequency?
3. For the superpower survey data, write a question that would be answered using a row conditional relative frequency.
4. For the superpower survey data, write a question that would be answered using a column conditional relative frequency.